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On the macroscopic formation length for GeV photons

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ABSTRACT

Experimental results for the radiative energy loss of 206 and 234 GeV electrons in 5–10 μm thin Ta targets are presented. An increase in radiation emission probability at low photon energies compared to a 100 μm thick target is observed. This increase is due to the formation length of the GeV photons exceeding the thickness of the thin foils, the so-called Ternovskii–Shul'ga–Fomin (TSF) effect. The formation length of GeV photons from a multi-hundred GeV projectile is through the TSF effect shown directly to be a factor 10^{10} longer than their wavelength.

1. Introduction

In the CERN NA63 experiment, we have investigated the influence from the formation length on radiation from a stack of thin foils, a so-called sandwich target. The formation length is due to the uncertainty in the longitudinal recoil momentum taken by the nucleus from which a high energy electron scatters during bremsstrahlung emission. From the uncertainty relations there is a corresponding distance over which the photon can be considered 'formed', i.e. the formation length. It is approximately equal to the distance of travel necessary for the electron to 'lag behind' the photon by one wavelength, i.e. loosely speaking the photon has 'separated' from the emitting particle. As shown below in Eq. (1), the formation length is approximately equal to $\simeq 2\gamma^2 c/\omega$ for soft photons where γ is the Lorentz factor of the emitting particle, c the speed of light and ω the angular frequency of the emitted photons. Even for GeV photons the formation length can be of macroscopic dimension if the energy of the emitting particle, and so γ, is large enough. For particle energies of around 200 GeV, the formation length for a 10 GeV photon becomes several microns long, i.e. in the experimentally accessible regime for the use of thin targets in high energy beams. The formation length — introduced more than 50 years ago by Ter-Mikaelian [1] — is fundamental to a number of phenomena in radiation physics, ranging from strong enhancements in crystals [2] to suppression effects in dense matter [3]. Furthermore, insights concerning formation lengths are applied in many other branches of physics, e.g. in the strong interaction [4], plasma-wave acceleration [5] or radiation from the electrosphere of strange stars [6]. For an in-depth coverage of the formation length phenomenon in radiation physics, the reader is referred to [7].

In the bremsstrahlung emission from an energetic positron or electron traversing a solid material, there are four basic scales of length: The radiation length, X_0, the foil thickness, Δt, the formation length

$$l_f = \frac{2\gamma^2 c(E - \hbar\omega)}{E\omega} \tag{1}$$

and the 'multiple scattering length', $l_\gamma = \alpha X_0/4\pi$, where $E = \gamma mc^2$ and $\hbar\omega$ are the energy of the emitting electron and photon, respectively, m is the mass of the electron and α the fine-structure constant. Of these lengths, the only one that depends on photon and particle energy is the formation length, whereas the other lengths depend on the target material or structure. We note, that

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in the non-relativistic limit, Eq. (1) yields a formation length equal to $2\lambda\!\!\!^{-}$, i.e. for the non-relativistic case the photon can be considered formed after a few reduced wavelengths. In other words, the investigation of macroscopic formation lengths requires extremely relativistic beams, $\gamma \gg 1$. The extent of the formation zone can also be viewed as due to the uncertainty of the region where a photon is emitted, originating from the impossibility to detect the exact point of origin with an angular detection resolution poorer than $1/\gamma$, the emission cone angle [2].

An alternative approach is to consider the 'semi-bare electron', following [8], i.e. the electric field of the emitting particle during the time immediately following a scattering event. Before the scattering event the electron and the electromagnetic field are co-moving whereas the time it takes the field to adjust to the new direction of the electron is finite: In the rest frame of the electron, the 'regeneration time' t' of the field at position $\vec{x}'$ is proportional to the distance from the electron to $\vec{x}'$. The time t' corresponds, by a Fourier transform, to an angular frequency $\omega' \simeq 1/t'$ (see e.g. [9]), where the primed coordinates here denote the rest system. Due to the Lorentz transformations to the frame of the laboratory, $t = t'\gamma$ and $\omega = 2\gamma\omega'$ the 'regeneration time' becomes $t \simeq 2\gamma^2/\omega$ which is the same as Eq. (1) in the soft photon limit, $\hbar\omega \ll E$.

1.1. Thin target — Ternovskii–Shul'ga–Fomin effect

The Ternovskii–Shul'ga–Fomin (TSF) effect of radiation emission in a thin target [10,11] is due to the formation length extending out of an amorphous target, and for a fixed energy of the projectile it therefore gets stronger the lower the photon energy. Ternovskii [10] was the first to consider the effect which was later treated in more detail by Shul'ga and Fomin [11–14]. It also has an analogue in the case of thin crystals [15]. In the TSF effect, the radiation yield is diminished compared to the Bethe–Heitler cross section, i.e. excluding the destructive interference arising from multiple scattering, but increased compared to the Landau–Pomeranchuk–Migdal (LPM) cross section which includes that effect. The LPM cross section has proven to be an accurate description of radiation probabilities in sufficiently thick targets at high energies [3,16–18]. In the LPM effect, the suppression appears if the formation length exceeds the multiple scattering length. Thus, it is possible to interpret an observation of the LPM effect as a measurement of the formation length, but in that case it is a measurement in units of the multiple scattering length. The latter quantity is not very well defined as it depends on conventions, e.g. it may be equal to $l_\gamma^* = 2l_\gamma = \alpha X_0/2\pi$ if defined as the length required for $\pm 2\sigma$ of the multiple scattering distribution to scatter an angle $1/\gamma$, see also [3] concerning conventions in the LPM effect.

For the Ternovskii–Shul'ga–Fomin (TSF) effect, the analysis is applicable for target thicknesses $l_\gamma \ll \Delta t < l_\mathrm{f}$, see e.g. [14]. Combining the formation length and the target thickness parametrized by $k_\mathrm{f} > 1$, $\Delta t = l_\mathrm{f}/k_\mathrm{f}$, the effect becomes appreciable for photon energies

$$\hbar\omega < \hbar\omega_{\mathrm{TSF}} = \frac{E}{1 + \frac{\Delta t}{2\gamma \lambda\!\!\!^{-}_c}} \tag{2}$$

where $\lambda\!\!\!^{-}_c = \hbar/mc$ is the (reduced) Compton wavelength.

The magnitude of the suppression factor can be evaluated from (see [19])

$$\kappa \simeq \frac{k_\gamma}{6(\ln k_\gamma - 1)} \tag{3}$$

where $\Delta t = k_\gamma l_\gamma$ and $k_\gamma \gg 1$ ensures $\Delta t \gg l_\gamma$. Our calculations of the TSF suppression factor shown in the figures below follow the more elaborate theory outlined in [14], whereas Eqs. (2) and (3) are useful for estimates only.

In the TSF case, the radiation emission probability becomes proportional to the logarithm of the target thickness instead of the usual linear relationship.

Direct measurements of the extent of the formation zone through the TSF effect have been done at comparatively low energies at SLAC [18]. In these experiments, among other things the case $l_\gamma < \Delta t < l_\mathrm{f}$ was studied. We here present a measurement of the extent of the formation zone through the TSF effect at photon energies two orders of magnitude higher, but also for the case $l_\gamma < \Delta t < l_\mathrm{f}$. We thus effectively measure the formation length of emitted GeV photons and hereby show that this can be of the order several microns.

In [19], the experiment was aimed at addressing formation zone effects for target thicknesses comparable to the multiple scattering length which in turn was smaller than the formation length, i.e. when $l_\gamma \simeq \Delta t < l_\mathrm{f}$.

Furthermore, in [19] multiple scattering dominated transition radiation (MSDTR) was shown to be a likely ingredient for an accurate description of the data, but at very low photon energies only ($<$ 4 GeV). Moreover, the excess of photons was substantial only for a target with 53 layers of Au, where the MSDTR is expected to be significantly higher than in the present case with a maximum of 20 layers. The contribution of MSDTR to the present experiment is therefore expected to be negligible apart from a small contribution below a photon energy of a few GeV.

2. Structured targets

In Blankenbecler's theory [20,21] of structured targets (see also [22–24]) interference mechanisms are considered for targets of up to 10 segments. It is shown that 'the photon spectrum is clearly developing a peak where the formation length is approximately equal to the distance between the centers of the plates' [20]. Even though these calculations are performed only for 25 GeV (and in a single case 50 GeV), we expect that this observation does apply to the general case. Therefore, Eq. (1) can be inverted setting the formation length l_f equal to the target spacing l_s leading to an onset of resonance at a photon energy

$$\hbar\omega_\mathrm{r} = \left(\frac{1}{E} + \frac{l_\mathrm{s}}{2\gamma^2\hbar c}\right)^{-1} \simeq \frac{2\gamma^2\hbar c}{l_\mathrm{s}}. \tag{4}$$

Such resonances could in principle be observed in the present experiment, but this depends sensitively on an accurate and constant positioning of each sub-target in the sandwich as discussed below.

3. Experiment

The experiment was performed in the H4 beam line of the CERN SPS using tertiary beams of electrons with energies of 206 and 234 GeV. In Fig. 1 we give a schematical overview of the active elements in the setup of the experiment. The incident electron beam is defined by a scintillator counter, S1, and the beam profile and angles are measured on an event-by-event basis in 3 drift chambers (DCs). In front of DC2 the target of about 2.6% X_0 is placed. The emitted photons are finally intercepted in a lead glass detector (LG), the low energy cut-off of which was set to 2 GeV in the analysis to avoid influence from the pedestal.

Four target configurations were used (identified below by the label given in parenthesis): (Ta5Al6) A 20 × 5 μm Ta 'sandwich'-target with 6 μm Al 'spacers' between each Ta foil, (Ta5Al8) A 20 × 5 μm Ta 'sandwich'-target with 8 μm Al 'spacers', (Ta10Al12.5) A 10 × 10 μm Ta 'sandwich'-target with 12.5 μm Al 'spacers' and a reference target (Ta100) consisting of a single foil of 100 μm Ta. For Ta, the multiple scattering length is $l_\gamma = 2.4$ μm. The background measured with an empty target has been subtracted from the data.

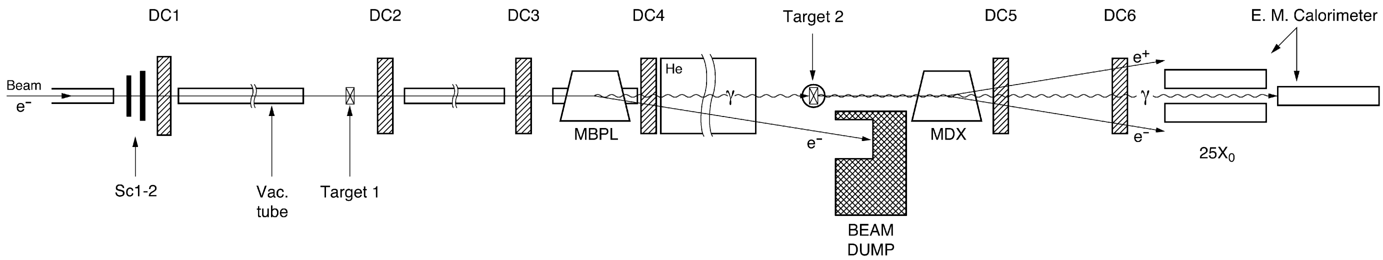

Fig. 1. A schematical drawing of the setup used in the experiment. The total length of the setup is about 65 m.

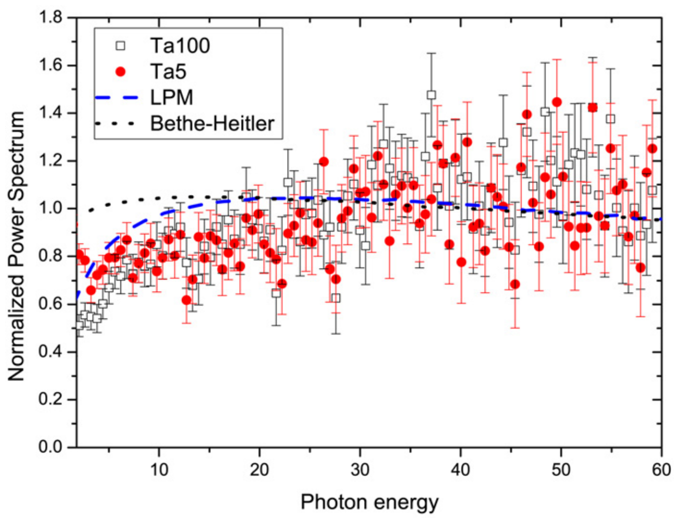

Fig. 2. (Color online.) Normalized bremsstrahlung power-spectrum, $\hbar\omega\, dN/d\hbar\omega \cdot X_0/\Delta t$, for 206 GeV electrons. The vertical scale is normalized to the number of incoming electrons and to the thickness in units of the radiation length. The filled circles show the Ta5 data set and the filled squares the Ta100 data set and theoretical values for a 100 µm Ta foil are shown by the lines: Bethe–Heitler (dotted) and LPM (dashed).

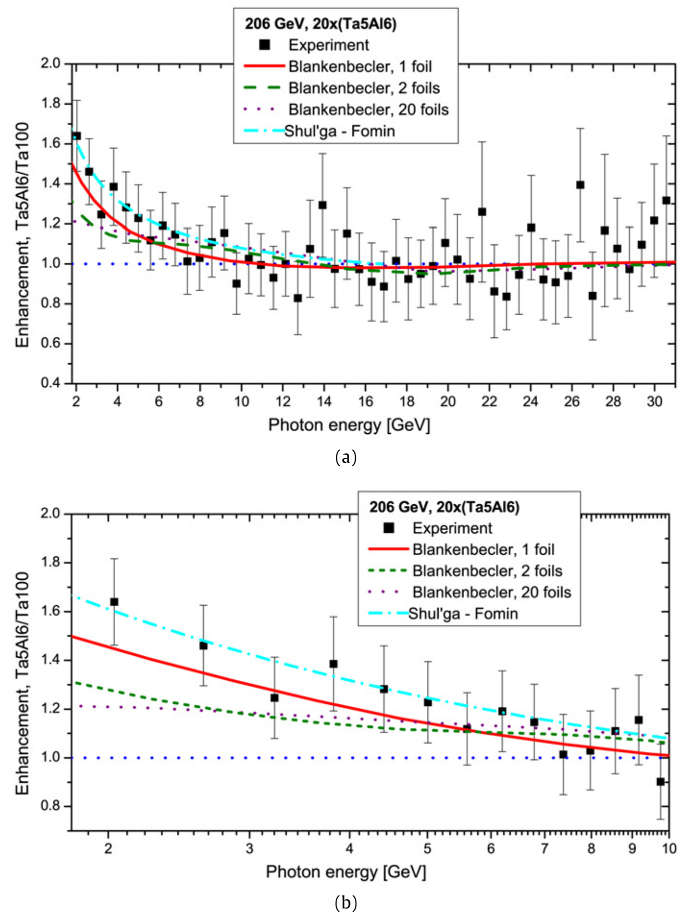

Fig. 3. (Color online.) (a) Enhancement of the normalized bremsstrahlung power-spectrum, $\hbar\omega\, dN/d\hbar\omega \cdot X_0/\Delta t$, for the Ta5Al6 target compared to the Ta100 reference target for 206 GeV electrons. The horizontal line shows the baseline of enhancement equal to 1, the full drawn line shows the expected enhancement from the calculations performed on the basis of the theory of Blankenbecler [20] for 1 foil, the dashed line for 2 foils and the dotted line for 20 foils. The dash-dotted line shows the theory of Shul'ga and Fomin [14]. In (b) is shown the same as in (a) but on a logarithmic scale, focused on the region of low photon energies.

The targets were mounted in a holder with an open area of 30 mm in diameter. Since the radiation length of Al is $X_0 = 89$ mm (22 times larger than $X_0 = 4.1$ mm for Ta), corresponding to a total of 0.4% X_0 in the Ta5 'sandwich'-target, the influence of the extra material thus introduced is marginal and the Al spacers can to a first approximation be treated as gaps. By the choice of number of sub-targets, each sandwich has approximately the same thickness in units of radiation lengths as the reference target Ta100, i.e. ratios of radiation probabilities are not expected to be significantly influenced by multi-photon emission effects.

4. Results

In Fig. 2 we show the radiation power-spectra at low energies for the Ta5 and Ta100 targets at 206 GeV, compared to theoretical values obtained by Monte Carlo simulation based on incoherent bremsstrahlung (Bethe–Heitler) and incoherent bremsstrahlung including multiple scattering (LPM). In the simulations, only one homogeneous radiator volume was assumed and the simulated spectra are based on the nominal value $E_{\mathrm{LPM}} = 3143$ GeV (LPM) and E_{LPM} set to 10^9 GeV (Bethe–Heitler). The LPM effect was incorporated in the framework of the GEANT package as described in [17,25].

The vertical scale has been normalized to the thickness expressed in units of the radiation length and to the number of incoming electrons. The thicknesses were evaluated by a least-squares fit using the Bethe–Heitler cross section for the central photon energies (22–45 GeV) with the thickness as a free parameter. A systematic difference of a factor 1.11 in the simulations between this single-photon spectrum and the simulated spectrum which includes pile-up is corrected for. The values found were in good agreement with the nominal thickness. There is a tendency for the data to lie below the simulated values at low photon energies. This could be due to acceptance or efficiency of the calorimeter. However, we emphasize that our main results given below consist in a comparison of the experimental data obtained from the sandwich targets to the experimental data for the Ta100 reference target, i.e. such systematics are eliminated as they affect both spectra equally.

In Fig. 3 we show the radiation probability for the Ta5 'sandwich' target at 206 GeV, compared to the Ta100 'reference' target,

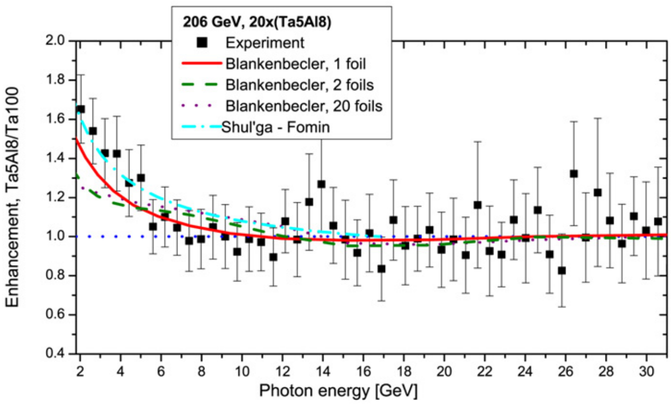

Fig. 4. (Color online.) As Fig. 3(a), but for the Ta5Al8 target for 206 GeV electrons.

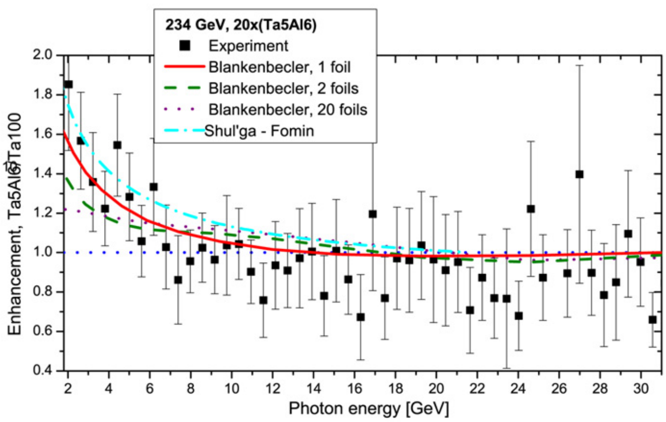

Fig. 5. (Color online.) As Fig. 3(a), but for the Ta5Al6 target for 234 GeV electrons.

i.e. an enhancement. In Figs. 3–6 the data points are based on experimental values only, i.e. all enhancements are given by the experimentally determined radiation probability for the 'sandwich' target in question, divided by the same experimental quantity for the Ta100 'reference' target. Below an energy defined by Eq. (2) $\hbar\omega_{\mathrm{TSF}} \simeq 12$ GeV where the TSF effect sets in for a 5 μm target, there is a clear excess of the radiation probability for the Ta5 'sandwich' target, compared to the Ta100 'reference' target, i.e. an enhancement larger than one. Similar results are obtained for the Ta5Al8 target as shown in Fig. 4. In Fig. 5 we show the enhancement for an electron energy of 234 GeV. As for the 206 GeV case there is a clear increase of enhancement at low photon energies. In all cases, the enhancement is consistent with 1 for photon energies from about 10 GeV up to the highest photon energy investigated (59 GeV).

Blankenbeclers theory of structured targets is based on the eikonal approximation and developed in collaboration with Drell [26]. In this theory the phase function of the electron is integrated along its path, leading to an 'alleviation' of the LPM effect, but never exceeding the normalized Bethe–Heitler yield (F always smaller than 1 [20, Eq. (25)]). Looking at Figs. 3–5 there is very good agreement between the enhancements observed in the experiment and those expected from Blankenbecler's theory for a single foil, but the agreement with the theory for the full set of foils is not as good. We suspect this to be due to an insufficiently accurate stacking of the sub-targets, i.e. the actual spacing is larger than the e.g. 6 μm of Al inserted. In that case, resonances arising from several targets are not to be expected at the photon energies investigated and the sandwich effectively acts as several independent foils. In other words, if each sub-target of 5 μm Ta

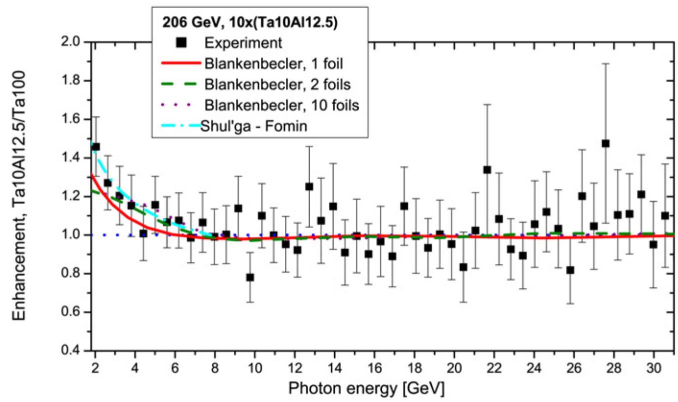

Fig. 6. (Color online.) As Fig. 3(a), but for the Ta10Al12.5 target for 206 GeV electrons.

acts independently in the sandwich due to the actual spacing being significantly larger than the nominal 6 μm, the TSF effect is dominant. This is further supported by the excellent agreement with the theory of Shul'ga and Fomin. Small imperfections in the spacers do not lead to large effects — a comparison of the Ta5Al8 and Ta5Al6 calculations for a sandwich of 20 foils, leads to differences less than 4%. Thus, the situation with $\Delta t < l_{\mathrm{f}}$ fulfilled for photon energies $\hbar\omega < \hbar\omega_{\mathrm{TSF}} \simeq 12$ GeV is shown to lead to an enhancement of up to 60%. As seen in Fig. 6, for the 10×10 μm Ta targets spaced by Al foils of 12.5 μm, there is a tendency for the data points to be in better agreement with the calculated values for the full target assembly than for the single foil. On the other hand, the agreement with the theory of Shul'ga and Fomin and the statistical uncertainties do not allow a firm conclusion as to the observation of a structured target resonance.

The increase in radiation probability for the targets at low photon energies can be attributed to the formation length of the photons extending out of each foil, i.e. by measuring the thicknesses of the sub-targets in the 'sandwich' targets, we effectively measure the formation length. Moreover, the theory of Blankenbecler for a single foil is in good agreement with that of Shul'ga and Fomin — both of which are in good agreement with measured values — but has the additional virtues of being applicable for all photon energies and several foils. On the other hand, the extraction of useful values for comparison with experiment is much easier using the latter theory than the former.

5. Conclusion

In conclusion, we have measured the formation length of GeV photons by bremsstrahlung of 206 and 234 GeV electrons to be several microns, in good agreement with the theories of Blankenbecler and Shul'ga and Fomin. We have thus experimentally observed the formation length of GeV photons directly — a length that surprisingly enough is macroscopic, 10^{10} times longer than the wavelength of the emitted photons.

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